



UNIVERSITY OF BASILICATA STUDIES
DEPARTMENT OF MATHEMATICS, INFORMATICS AND ECONOMICS

COURSE: Functional Analysis			
ACADEMIC YEAR: 2019-2020			
TYPE OF EDUCATIONAL ACTIVITY: Free choice			
TEACHER: ANGELICA MALASPINA			
e-mail: angelica.malaspina@unibas.it		website:	
phone: +390971205879		mobile (optional):	
Language: italian			
ECTS: 6	n. of hours:48	Campus: Potenza Dept.: DiMIE Program: Mathematics	Semester: first

EDUCATIONAL GOALS AND EXPECTED LEARNING OUTCOMES

The course aims at analyzing the basic arguments of functional analysis in Banach spaces. At the end, students should be able

- to know the basic elements of functional analysis, the theorems and the main techniques of the theory of linear and continuous operators, Banach spaces and Hilbert spaces,
- To own skills to solve various exercises,
- To read and understand texts of Functional Analysis,
- To provide themselves a mathematical proof of simple statements, with strong reasoning skills,
- To communicate in Italian the mathematical knowledge acquired in the course, as well as related issues.

PRE-REQUIREMENTS

The main items required are some knowledge of measure and integration theory together with a good background in point set topology.

SYLLABUS

Metric spaces.

Hölder inequality and Minkowski's inequality for finite sums. The spaces l^p and l^∞ . Separability, completeness. The Baire category theorem. Principle of uniform boundedness for metric spaces.

Banach Spaces.

Normed spaces: definition and examples. Metrics inducing a norm. Banach spaces. The analytic form of the Hahn-Banach theorem (real and complex cases). The geometric forms of the Hahn-Banach theorem: separation of convex sets. Linear and continuous operators between normed spaces. The norm of a linear operator, the space $B(X,Y)$. The dual space. The Hahn-Banach theorem for normed spaces and its consequences. The adjoint operator. Theorem related to the study of the functional equation $Tx = b$. Banach-Steinhaus theorem. The open mapping theorem and the closed graph theorem. $L^p(X,\mu)$ spaces. $L^\infty(X,\mu)$ space. The Riesz representation theorem in L^p . Reflexive spaces. Invertible operators in $B(X)$. Spectrum of an operator in $B(X)$. Sufficient condition for invertibility for an operator in $B(X)$. Neumann series. Compactness of the spectrum.

Hilbert spaces.

Space with scalar product. Elementary properties. Hilbert spaces. Generalization of the Pythagorean Theorem. Gram-Schmidt orthonormalization theorem. Bessel's inequality. The Riesz-Fisher theorem. Orthonormal bases. Fourier series in Hilbert space. Parseval's equality. Characterization theorem of Hilbert spaces. Projections. Continuous linear functional on a space with scalar product. The representation theorem of Riesz-Frechet and its consequences. The Hahn-Banach theorem for Hilbert spaces. Weak convergence.



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TEACHING METHODS

The course includes 48 hours of frontal lectures. The main theorems are shown and examples and applications are provided. Some blackboard exercises are proposed to students in order to train them.

EVALUATION METHODS

The course evaluation consists in an oral examination.

TEXTBOOKS AND ON-LINE EDUCATIONAL MATERIAL

- E. Giusti: *Analisi Matematica 2*, Bollati Boringhieri, 1983.
- R. E. Megginson: *An introduction to Banach space theory*, Springer, 1998.
- H. L. Royden: *Real Analysis*, Collier Macmillan, 1988.
- B.V. Limaye: *Functional analysis*. New Age international, 1996.
- S. Salsa: *Equazioni alle derivate parziali*, Springer, 2007
- Teacher's handouts.

INTERACTION WITH STUDENTS

Student office hours: tuesday and wednesday from 11:30 to 13:30.

Students can also contact the teacher by sending an email to angelica.malaspina@unibas.it

EXAMINATION SESSIONS (FORECAST)¹

06/02/2020, 05/03/2020, 09/06/2020, 07/07/2020, 17/09/2020, 15/12/2020.

SEMINARS BY EXTERNAL EXPERTS YES NO

FURTHER INFORMATION

¹ Subject to possible changes: check the web site of the Teacher or the Department/School for updates.