Università degli Studi della Basilicata
Dipartimento di Matematica, Informatica ed Economia

COURSE: Topics in Approximation Theory			
ACADEMIC YEAR:2019-20			
TYPE OF EDUCATIONAL ACTIVITY: Basic			
TEACHER: RUSSO Maria Grazia			
e-mail: mariagrazia.russo@unibas.it		website: Google Classroom (code: wbbe53e)	
phone: 0971205147		mobile (optional): 3204235379	
Language: italian			
ECTS: 6	n. of hours: 48	Campus: Potenza Dept./School: DiMIE Program: Mathematics (Master degree)	Semester: II

EDUCATIONAL GOALS AND EXPECTED LEARNING OUTCOMES

The teaching of Topics in Approximation Theory is an advanced modulus of Numerical Analysis. It is strongly related to the numerical resolution of linear integral equations by means of methods based on the polynomial approximation, also in the bivariate case. The main topics are:

• Polynomial approximation in one and two dimensions.
• Mapping properties, in suitable weighted functional spaces, of the integral operators appearing in Fredholm and Cauchy singular integral equations, both in the univariate and bivariate case.
• Collocation and Nyström methods for the involved integral equations.
• Convergence, stability and well conditioning of the proposed methods in weighted spaces of functions.

The main skills will be:

• Develop critical thinking regarding the choice between antagonistic methods for solving a specific integral equation (eg. comparing the speed of convergence, stability of algorithms, the memory footprint, the computational cost).
• Achieving a good level of familiarity in the individual programming of numerical algorithms in Matlab.
• Knowing how to interpret the numerical data from the computer and know how to evaluate consistency with the expected results.

PRE-REQUIREMENTS

Are prerequisites:

• the basic topics of numerical analysis and in particular the concepts related to: the finite arithmetic, conditioning of a problem, stability of an algorithm, numerical solution of system of linear equations, quadrature formulas, polynomial approximation ;
• the basic concepts of the Functional Analysis: normed spaces, orthogonal systems, operator sequences, Fredholm Alternative.
• a basic procedural programming and knowledge of the Matlab language.

SYLLABUS

1. **Basics on the polynomial approximation (2 hours):**
   References to the basic concepts of the polynomial approximation, also in the weighted case. References to the interpolating Lagrange operator. References to the orthogonal polynomials. Fourier operators in weighted spaces.
2. **Polynomial approximation of bivariate functions (4 hours+ 2 hours of exercises):**
   Polynomial approximation in two dimensions. Bivariate best approximation. Fourier and Lagrange operators, as tensor products. Convergence and stability.
3. **Cubature formulas (4 hours+ 2 hours of exercises):**
   Cubature formulas. Gaussian cubature formula. Non tensorial cubature formula based on *Padua points.*
4. **Numerical methods for Fredholm integral equations on the square (4 hours+ 2 hours of exercises):**
   Mapping properties of the integral opertors in weighted spaces of functions. Solvability of the integral equations in such spaces. Nyström and collcoation methods based on zeros of orthogonal polynomials.

5. **Numerical methods for Fredholm integral equations defined on unbounded domains of the plane (4 hours+ 2 hours of exercises):**
   References to the polynomial approximation on unbounded subintervals of the real line. Mapping properties and solvability of the Fredholm integral equations defined on inbounded domains of the plane. Nyström methods based on the zeros of orthogonal polynomials and the truncation strategy..
6. **Cauchy singular integral equations (10 hours + 4 hours of exercises):**
   Study of the Cauchy singular integral equations defined on [-1,1] in L^2 weighted spaces and in spaces of weighted continuous functions. Mapping properties of the involved integral operators. Direct and indirect methods based on the polynomial approximation. A Nyström method. Cauchy bisingular integral equations.
7. **Iterated Bernstein polynomials (6 hours + 2 hours of exercises**):
   References of the Bernstein polynomials. Bernstein iterated polynomials in one and two dimensions. Quadrature and cubature formulas based on the iterated Bernstein polynomials. Nyström methods for Fredholm integral equations in one or two dimensions, based on the Bernstein iterated polynomials.

TEACHING METHODS
Theoretical lessons, Laboratory tutorials

EVALUATION METHODS
The aim of the examination is to test the level of achievement of the above mentioned educational goals.

The test is oral.

TEXTBOOKS AND ON-LINE EDUCATIONAL MATERIAL
Notes are available on the web site of the course

Textbooks:

1. M. C. De Bonis, G.M. Mastroianni, I. Notarangelo, Elementi di Teoria dell'approssimazione polinomiale, Aracne Editrice
2. K.E. Atkinson, The Numerical Solution of Integral Equations of the Second Kind, Cambridge University Press

INTERACTION WITH STUDENTS
During the first lesson of the course the objectives, the program, the verification methods and all information related to the operation, including the description of the web page of the course are described.
Access to the course website, which is part of an e-learning platform (Classroom of Google Suite) is open for the student of the course and contains, in addition to all the material used during the course, also a messaging facility that allows the teacher to communicate directly with students and vice versa.

Weekly office hours: Thursday from 15.30 to 17.30 at the office of teacher (3D-building room 216)
In addition to weekly reception, the teacher is available through its e-mail (mariagrazia.russo@unibas.it), phone (3204235379), and the aforementioned News Forum of the course web site. Moreover she receives by appointment on different days from Thursday.

EXAMINATION SESSIONS (FORECAST)[1]
06/03/2020, 15/05/2020, 08/06/2020, 20/07/2020, 14/09/2020, 18/12/2020

SEMINARS BY EXTERNAL EXPERTS YES □ NO x

FURTHER INFORMATION

[1] Subject to possible changes: check the web site of the Teacher or the Department/School for updates.

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